<u>SPECIAL RELATIVITY:</u> The Physics of High Speeds

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CHAPTER II

LIGHT AND THE ETHER

DURING THE NINETEENTH CENTURY, as a means of trying to understand light-waves, it was believed by many physicists that the universe was filled with a substance called "ether." There were at least two excellent reasons to believe in the existence of ether. First of all, it was felt that all waves require something to "wave" in; water waves need water, sound needs air, etc. Light then must wave in "ether," because something is needed to provide the restoring force necessary to maintain oscillations. Since light waves travel very well through a vacuum, the ether hypothesis provided a means other than ordinary material media for supporting these oscillations.

The second reason for believing in the ether was more convincing, and very hard to get around. The ether at rest defined that coordinate system in which light travels with its characteristic velocity, c. This means that if you happen to be moving with respect to the ether, a beam of light will seem to move with velocity either less than c or greater than c, depending on whether you move with the light or against it. This is what is found with other kinds of waves. Sound moves with its characteristic speed with respect to the air. If a wind is blowing, sound will travel faster than usual going downwind, and slower than usual going upwind. This is built into the Galilean transformation given in Chapter I.

So people began, during the latter part of the 19th century, to try to detect the ether. Of particular interest was the question: is the ether at rest with respect to the earth, or is it moving? How fast is the ether wind blowing past us? We might expect off-hand a seasonal variation brought about by the changing direction of the earth's velocity around the sun. Also it would seem possible that the ether might blow stronger on mountain tops, where the earth has less chance to impede the flow. That is why ether experiments were done at different times during the year, and why many were done on mountain tops.

A. The Aberration of Light

A certain effect, known as the aberration of light, was of importance in the ether investigation. This effect had been known ever since 1727, when Bradley* observed that the stars seem to perform an annual circular motion in the sky. This apparent motion was understood to be due to the fact that the observed direction of a light ray coming from a star depends on the velocity of the earth relative to the star. The angular diameter of the circular orbits is about 41 seconds of arc, which can be understood by a consideration of Figure 2.1. Because of the motion of a telescope during the time it takes for light to travel down the length of the tube, the light will appear to follow a path which is tilted with respect to the actual path.

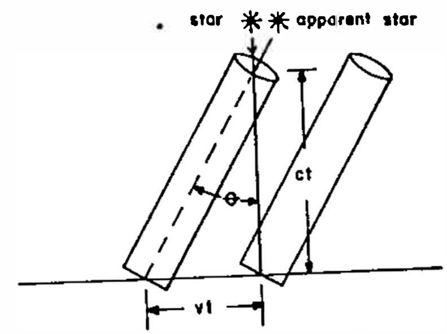

Figure 2.1

The figure shows a star which is straight overhead, being viewed through a telescope. The telescope is mounted on the earth, which is moving to the right with velocity v in the course of its orbit about the sun. The light requires a finite time t to travel down the length of the tube, in which time it covers a distance ct, as shown in the figure. But during this time the telescope has moved to the right a distance vt, as shown. Therefore if the light ray is to strike the eyepiece at the bottom of the tube (rather than the side of the tube), the telescope has to be tilted to the right. The star itself then appears to be in a different position, at an angle θ from the vertical. Six months later, at a time when the star is actually again overhead, the telescope will have to be tilted to the left in order to see the star. More generally, as the earth revolves about the sun the telescope has to be continuously adjusted so as to point slightly in the direction of the earth's motion. Thus as the earth circles the

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* J. Bradley, Phil. Trans. 35, 637 (1728). See also reference 1.

sun, the star will appear to move in a small circular path. From the figure, which displays a grossly exaggerated angle θ, it is seen that $\tan\theta = v/c$. Using $v = 30$ km/sec for the velocity of the earth in its orbit, we have

$$\tan\theta \cong \theta = \frac{30 \times 10^3 \text{ m/sec}}{3 \times 10^8 \text{ m/sec}} = 10^{-4} \text{ radians} \tag{2-1}$$

which is about 20.5 seconds of arc. This is in excellent agreement with the observed value (of the circle's radius), as previously quoted.

We conclude from these observations that the ether is not dragged around with the earth. If the ether were at rest with respect to the earth, the telescope could be pointed vertically, so there would be no aberration effect. The ether in Figure 2.1 would then be moving to the right with velocity v, pulling the light ray with it (just as a wind pulls sound with it), so there would be no need to correct for the earth's motion by tilting the telescope. In short, if there is an ether, it must be blowing past us at an average of at least 30 km/sec! Obviously it would be advisable to perform some experiment to see if this is really true!

B. The Michelson-Morley Experiment

In 1887, Michelson and Morley performed a very sensitive experiment in an attempt to detect the motion of the ether. By looking for effects of light interference, they hoped to measure the velocity of the ether wind. The apparatus used was the Michelson interferometer, as shown in somewhat simplified form in Figure 2.2. B and C are fully reflecting mirrors, and A is a half-silvered mirror which reflects half and transmits half of the light incident on it. The idea is this: light from the source strikes A, half of it being reflected up toward B, and the other half transmitted through to C. The light striking mirror B is reflected back, and half of it is transmitted through A to the observer O. The light striking the right-hand mirror C is reflected back, and half of it is reflected off A to observer O. Altogether, half of the light leaving the source reaches the observer, of

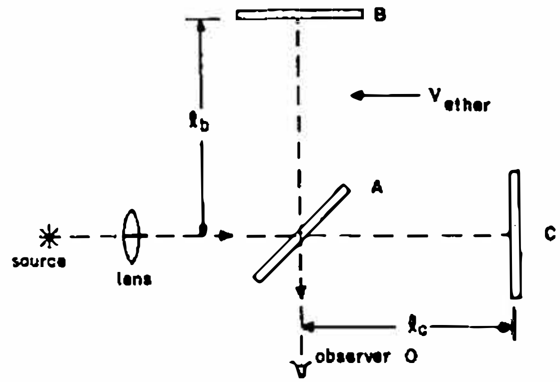


Figure 2.2

which half follows the path ABAO, and half the path ACAO. Since these two paths are generally of different length, the light waves reaching the observer from the path ACAO will be out of phase with those from the path ABAO, so the observer will see interference effects.* The fact that the wavelength of visible light is so small (about 5×10^{-5} cm.) means that there will be a rapid alternation of constructive and destructive interference as the relative path-lengths are changed, an indication that the interferometer is a very sensitive device.

What does the ether have to do with this experiment? Suppose we have adjusted the paths ABAO and ACAO to be of exactly equal length, and suppose the ether is sweeping past the apparatus from right to left with (unknown) velocity v, as shown in Figure 2.2. Then, in going from A to C, the light will have to fight upstream against the ether current, while going back from C to A it will be swept back with the current. The light going from A to B and back to A will be moving largely cross-current, although it will have to fight somewhat against the current or it would be swept downstream and never return to A. We shall see that even if the two paths are of the same length, it takes longer to swim upstream and downstream than to swim cross-

* A brief introduction to the interference of light-waves is given in Appendix H.

current, so the time intervals will be different, and interference can take place.

Recalling that we've assumed light moves with velocity c with respect to the ether, just as sound travels with its characteristic velocity with respect to the air, we can calculate the time needed to traverse each path. In going upstream from A to C, the light will travel at speed c-v with respect to us, and so requires a time $\ell/c-v$. Going downstream from C to A, the speed is $c + v$, so the time required is comparatively short, $\frac{\ell}{c+v}$. The total time for the trip ACA is therefore

$$\frac{\ell}{c-v} + \frac{\ell}{c+v} = \ell \frac{2c}{c^2 - v^2} \text{ or finally } t_{ACA} = \frac{2\ell}{c} \frac{1}{1 - v^2/c^2}. \qquad (2\text{-}2)$$

The time t_{ABA} for the cross-current trip is most easily calculated in the ether's frame of reference, as shown in Figure 2.3. In this frame the ether is at rest, and the apparatus moves to the right with velocity v. During the time $\frac{t_{ABA}}{2}$ the light takes to travel from A to B, the light moves a distance $\frac{ct_{ABA}}{2}$, and the apparatus moves a distance $\frac{vt_{ABA}}{2}$.

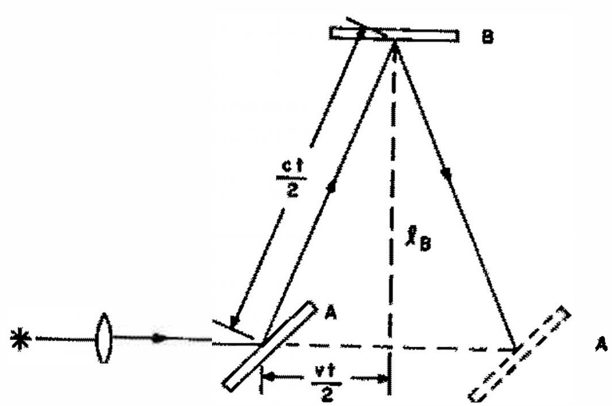


Figure 2.3

Ether at rest, interferometer moving to the right with speed $\underline{v}$.

From the rule of Pythagoras, we have

$$\ell^2 + (\frac{vt}{2})^2 = (\frac{ct}{2})^2 \text{ or } t^2 (c^2 - v^2) = 4\ell^2$$
$$\text{or } t_{ABA} = \frac{2\ell}{c} \frac{1}{\sqrt{1 - v^2/c^2}}. \qquad (2\text{-}3)$$

Comparing with the upstream-downstream time t_{ACA}, we see that

$$t_{ACA} = \frac{t_{ABA}}{\sqrt{1 - v^2/c^2}} \qquad (2\text{-}4)$$

so it takes longer to go up and downstream than to go sideways. Since we expect that v<<c, it takes a very sensitive device to tell the difference. The difference in time means that the two light-beams will be out of phase with each other and consequently will produce interference patterns, even if the path lengths are just the same.

Now in fact with a single measurement it is not possible to separate the effect of different path lengths from the effect of the ether wind. Therefore, it is necessary to look at the interference fringes in one position, and then rotate the apparatus by 90° to interchange the position of the interferometer arms with respect to the ether wind. During the second measurement the path ACA will be cross-current, and path ABA will be upstream and downstream.

Michelson and Morley's actual apparatus allowed multiple reflections so as to increase the path length. The optical system was mounted on a heavy sandstone slab, which was supported on a wooden float, which in turn was designed to float in a trough containing mercury. This made turning easy and smooth, and reduced the effects of vibration. The effective optical length of each arm of the interferometer was about 1100 cm., which would theoretically lead to a shift of 0.4 fringes when the apparatus was rotated, assuming the ether wind was about the same as the orbital speed of the earth. From very careful measurements in July of 1887, they concluded that "if there is any displacement due to the relative motion of the earth and the luminiferous ether, this cannot be much greater than 0.01 of the distance between the fringes."*

The simple conclusion from the null result of Michelson and Morley is that the ether is not blowing past us. It must be nearly stationary relative to the earth. It seems incredible that the earth should be so favored, although conceivably there might be a drag effect sufficient to drastically reduce the wind near the earth's surface. But then we are in trouble with the aberration of light. From this effect, as previously discussed, it was concluded that the ether could not be dragged around with the earth. An ether wind of velocity equal to the earth's orbital velocity was needed to explain the small annual circular motion of the stars.

Many other experiments contributed to the confusion surrounding the influence of the ether, such as experiments with light shining through moving water, and measurement of magnetic forces between charged capacitor plates. Naturally several explanations were advanced, some of them very interesting and clever, but none successfully explained all the experimental results.

REFERENCES

1. An interesting account of Bradley's experiments with aberration is contained in "The Discovery of Stellar Aberration" by Albert B. Stewart in the Scientific American, March 1964, p. 100.

2. Two articles on the history of the Michelson-Morley experiment by R. S. Shankland are in the American Journal of Physics 32, p. 16, 1964, and the Scientific American, December 1964, p. 107.

3. Other papers on the experimental basis of relativity are referred to by Panofsky and Phillips in Chapter 14, Classical Electricity and Magnetism (Addison-Wesley, 1955).

* See reference 2.

4. E. T. Whittaker has written a history of the ether, entitled A History of the Theories of Aether and Electricity (Thomas Nelson and Sons, 1951).

5. A biography of Michelson entitled Michelson and the Speed of Light has been written by Bernard Jaffe (Anchor Books, Doubleday & Co., 1960).

PROBLEMS II

1. An artificial earth-satellite completes a 26,000 mile orbit in 90 minutes. Find the angle subtended by the radius of the circle covered by a star as seen from the satellite, due to light aberration.

2. A river is 300 feet wide and flows at 1 foot/second. Two swimmers can each swim at 2 feet/second. One swimmer swims downstream 300 feet and then swims back upstream to where he began, as seen from the shore. The other swims straight across the river and back to where he started on the original shore. Find the time required for each to complete his trip, and verify Eq. 2-4. Remember that the speed of each swimmer is 2 feet/second only in the water's frame of reference.

CHAPTER III

EINSTEIN'S POSTULATES

AS EVIDENCED by many experiments, including light aberration and the results of Michelson and Morley, the ether wind cannot be detected. If there is an ether, it apparently has no influence on physics. No measurements ever made have indicated that it makes the slightest difference what the ether is doing – whether it is at rest or blowing past us. Therefore since we don't observe it, it seems reasonable to discard the idea that it exists. This seems easy to do, and we wonder what all the excitement was about – until we recall that we don't know the frame of reference in which light travels with speed c! The ether frame was supposed to be that frame, but now we don't have it. The ether has vanished along with its frame.

There seems to be no way to find the frame in which light moves with speed c. Various suggestions were put forward: for example, might it not be that light travels at speed c with respect to the source which emits it? The idea that this is the special frame was contradicted by observing the light from double stars. For if at a particular time one star is approaching and the other receding from us as they orbit around one another, light would reach us from the approaching star first. A detailed analysis of this effect shows that the double star system would appear to behave in a different way than is actually observed.* (Recent experiments with elementary particle decays appear to provide more conclusive evidence against this "emission theory," as described in reference 4.)

It is here that Albert Einstein appeared on the scene. While working in a Swiss patent office, he published a paper in 1905** which set forth the basis of what he later called the special theory of relativity. The theory is founded on two rather innocent-sounding postulates, which are that

* See reference 3.

** "Zur Elektrodynamik bewegter Körper" (On the Electrodynamics of Moving Bodies), Annalen der Physik 17, 1905.

1) Absolute uniform motion cannot be detected.
2) The velocity of light does not depend on the velocity of its source.

Neither statement seems particularly upsetting, but the combination of the two is revolutionary. The first postulate says that motion in a straight line at constant speed cannot be detected – merely meaning that there is no absolute frame with which all motion can be compared. All velocities are relative. There is no "absolute space" or "ether frame" which is at "rest." All we can measure is the velocity of an object in relation to another object. This idea that no inertial frame is to be preferred above any other for viewing physics is certainly not original with Einstein, but is a reaffirmation of the same assumption implicit in Newton's laws, as discussed in Chapter I.

The first postulate implies that the laws of physics must look the same in any inertial frame. If they varied, one frame could be singled out as being fundamentally "better" than another (say because of greater simplicity of the laws), so could become the preferred frame with respect to which all velocities should be measured. We should stress that it is uniform motion which can't be detected, since it is usually easy to tell whether or not you are accelerating, as by watching the behavior of a pendulum or a spring with a mass on the end.* The special theory of relativity deals only with measurements made in inertial frames of reference.

* As a matter of fact, if an observer with such "accelerometers" is accelerating at a constant rate, the behavior of the accelerometers will be the same as similar ones in an inertial frame set in a uniform gravitational field. The pendulum will swing and the spring with the mass attached will stretch or compress depending upon how it is oriented. Conversely, such an accelerometer in free fall in a uniform field will give no reading of acceleration at all. As a step in the development of the so-called "general theory of relativity" of 1915, which is a theory about gravity, Einstein postulated that in a restricted region of space no experiment can distinguish between a uniformly accelerated frame of reference and a uniform gravitational field. This postulate is one form of the "Principle of Equivalence," which is further discussed in Appendix D.

Since the first postulate is not particularly disturbing, and even seems quite plausible, let us proceed to the second postulate. Denying the idea mentioned previously, it says that "the velocity of light does not depend on the velocity of its source." Some other things in physics have this property, and some do not. For example, if we stand on the sidewalk watching a car go by, and somebody in the car throws a rock straight ahead, the rock's velocity with respect to us will depend on the velocity of the car. In fact, as everybody knows (at least within the limits of experimental error)

$$V_{rock,\ us} = V_{rock,\ car} + V_{car,\ us} \tag{3-1}$$

which is just the additive law of velocities. Therefore rocks don't obey the second postulate.

On the other hand, consider the motion of sound in air. It travels at about 1100 ft/sec with respect to the air. Therefore an observer will find that the measured speed of sound has nothing to do with the velocity of the sound-source through the air. Two people in front and in back of a moving car, at equal distances from it when it honks its horn, will hear the honk at the same time. So as long as the velocity of the sound-source is measured with respect to the air, sound is like light in that it obeys the second postulate. The sound velocity is independent of the motion of its source.

But suppose we decide to measure the velocity of the sound-source with respect to the observer. That is, the observer measures the velocity of sound and of the sound-source with respect to himself. Then the situation is quite different. For the speed of sound clearly does depend on the motion of the observer through the air. An observer moving through the air toward a sound-source will measure a sound velocity of greater than 1100 ft/sec, and an observer moving through the air away from the source will measure a sound velocity of less than 1100 ft/sec. Therefore obviously the speed of sound does depend on the source velocity if this source velocity is measured with respect to the observer. Sound obeys Einstein's second postulate in only one special frame of reference, in which the observer is at rest in the air.

The crucial difference between sound and light is then immediately clear. Since there is no ether (which would correspond to the air in the case of sound), light has to obey the second postulate in all inertial frames. Without the ether there is no preferred frame to be chosen above any other. The speed of light cannot depend on the source velocity regardless of the reference frame of the observer. It is this fact which produces the first surprise of relativity. Imagine a searchlight out in the middle of empty space, which sends out a continuous beam of light. Some distance away are two spaceships, one at rest with respect to the searchlight and the other moving toward it at relative velocity $c/2$, as shown in Figure 3.1. Observers in both ships are equipped to measure the velocity of light from this searchlight. The "stationary" observers

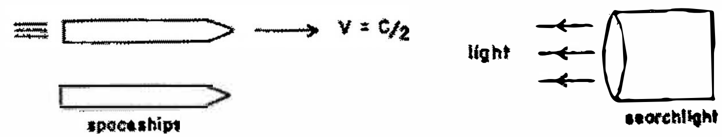
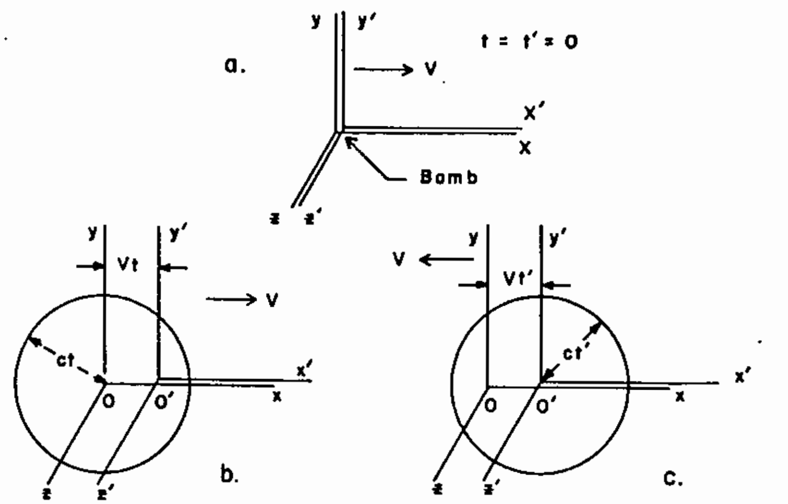


Figure 3.1

will of course measure the velocity to be its standard value of $c = 3 \times 10^8$ m/sec. On the basis of intuition (e.g. from rocks and cars) we might then say that the light velocity measured by the "moving" observers will be $c + v = c + c/2 = 3/2 \times 3 \times 10^8$ m/sec. But then we contradict the postulates of Einstein! For the first postulate states that the situation with the spaceship racing toward the searchlight is exactly the same as if the searchlight were racing toward a stationary spaceship (who can tell which is moving?). But then the second postulate claims that the measurement of light velocity in this latter case must give the same result as if the searchlight source were not moving toward the spaceship. But this result would be just $v_{light} = c$! Our guess of $v_{light} = 3/2\ c$ for a moving observer was wrong, and should have been $v_{light} = c$. It takes both postulates to force this conclusion. Therefore the velocity of light is independent of the observer's motion. It is the same in every inertial frame of reference. This is a revolutionary idea, unprecedented before Einstein. It took considerable nerve to write down postulates which had as a consequence that light always goes at the same speed no matter how fast the observer is moving.

A particular consequence of the constancy of light's velocity is the following: imagine two sets of rectangular coordinates (inertial frames) which are moving with uniform relative velocity V. At a certain time, the origins of the two systems pass each other, and a bomb explodes at the point where the origins instantaneously coincide, as shown in Figure 3.2. The light from the flash of the explosion will spread out in all directions, the wave-front forming a sphere of radius ct. Observers in one of the two frames will note that the center of this expanding sphere is at the origin of their frame, which is where the bomb went off. But observers in the <u>other</u> frame will find that the center of the sphere is at the origin of <u>their</u> system of coordinates, since the light left that spot at t = 0 and spread out in all directions at the same velocity. In other words, the wave-front forms a sphere in both frames of reference, and the observers in each frame claim that the center of the sphere is at <u>their own</u> origin of coordinates! This appears to be paradoxical, since after t = 0 the origins don't coincide. Yet the conclusion is forced by the two postulates.



A time t later as seen by observers in the <u>unprimed</u> frame.

A time t′ later as seen by observers in the <u>primed</u> frame.

Figure 3.2

In order to actually perform this experiment, <u>several</u> observers are needed in each frame. A <u>single</u> observer can't stand back and watch a sphere of light expand. One observer should be located at the frame origin to verify that the explosion happened there at time t = 0. Others can be stationed here and there with synchronized clocks and a knowledge (from measurements with meter-sticks) of how far they are from the origin. If each receives the light flash at time t = r/c, where r is his distance from the origin, they can all compare notes afterwards and be satisfied that the light spread spherically from the origin of their own frame.

So <u>light</u> from the explosion spreads out in such a way that observers in each frame conclude that it spreads spherically, and is centered about <u>their own</u> origin. But how about <u>sound</u> from the explosion? Sound moves with its characteristic velocity only in the frame in which the air is at rest. Only in <u>that frame</u> (say the unprimed frame) will the sound expand spherically with its center at the frame origin. In a primed frame, the sound will spread spherically, all right*, but the sphere center will always be located at the origin of the <u>unprimed frame</u>. So as you might expect, to primed observers the expanding sphere will drift steadily with a velocity equal to the wind velocity felt by them due to their motion through the air. Thus again the great difference between the behavior of sound and light in different frames of reference can be traced to the absence of an ether frame for light.

We've only begun to explore the consequences of Einstein's postulates. Further investigation of time and distances will help to explain how expanding light-spheres can behave in such a paradoxical fashion. From the results of the next three chapters, Appendix F will show how this apparent paradox can be understood.

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* Approximately, for sound velocities much less than the speed of light. See Chapter V.

REFERENCES

1. Einstein's original paper on special relativity is "Zur Elektrodynamik bewegter Körper" in the journal Annalen der Physik 17, 1905. A translation is available in the paperback "The Principle of Relativity" by Einstein and others (Dover Publications, 1923).

2. Interesting accounts of how Einstein thought of relativity are contained in his autobiographical notes in Albert Einstein: Philosopher-Scientist, Vol. I, edited by P. A. Schilpp (Harper Torchbooks, 1959) and in a fascinating account of "Conversations with Albert Einstein" by R. S. Shankland in the American Journal of Physics 31, 47 (1963).

3. Reports and discussion on double star data are in articles by Comstock in Phys. Rev. 30, 267 (1910) and by deSitter in Proc. Amsterdam Acad. 15, 1297 (1913) and 16, 395 (1913), and are reviewed in reference 4 below.

4. A review of old and new experiments relating to emission theories and Einstein's second postulate is in an article by J. G. Fox in the American Journal of Physics 33, 1 (Jan. 1965).

PROBLEMS III

1. Do water-waves obey a second postulate in
   a. some frame?
   b. all frames?

2. Devise a way for observers in a given frame to verify experimentally that light spreads out spherically and is centered about their origin, as in the example mentioned in the chapter. Devise a scheme for measuring the shape of the sound wave-front as well.

3. From our conclusion that the speed of light is the same in all frames, show that the analysis of the Michelson-Morley experiment in Chapter II is in error. Show also that if Einstein is correct there should be no fringe shift in the experiment.

CHAPTER IV
TIME DILATION

"until at last it came to me that time was suspect" – A. Einstein

THE PHENOMENON of time dilation follows from the result of Chapter III that light travels at the same speed in all inertial frames. The term "time dilation" means that moving clocks run slow. That is, we will show that if we were to compare the readings of moving clocks with the readings of similar clocks at rest in our own frame of reference, we would find that the moving clocks run behind in time.

To demonstrate this effect, suppose we are sitting in a spaceship in the middle of empty space, and watch another ship go by at some velocity v, as shown in Figure 4.1. On the other ship are two men A and B, across from each other as shown. Each of the two has a clock, synchronized with the other, and the distance between them has been previously measured to be d. Man A suddenly explodes a flashbulb when his clock reads zero, and B measures the time at which the light-flash reaches him. He finds of course that this is $t' = d/c$, since that is what is meant by saying that light travels at speed c. Note that t' is the time interval measured by two clocks on the spaceship.

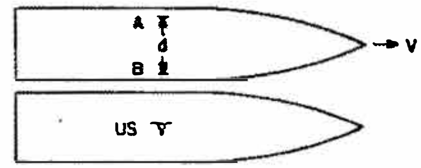


Figure 4.1

Now consider what this sequence of events would look like in our frame of reference. To us the ship moves somewhat during the time the light is traveling between A and B, so that to us the light has to go farther than d in order to reach B. In fact, in our frame the light moves along the hypotenuse of a right triangle, as shown in Figure 4.2. One leg of this triangle is the distance d, and the other leg is the distance the ship moves while the light is traveling. The fact that the light moves at an angle in our frame of reference is just the aberration of light effect discussed in Chapter II. A ball thrown from A to B would also move at some angle in our frame, although this angle would be much

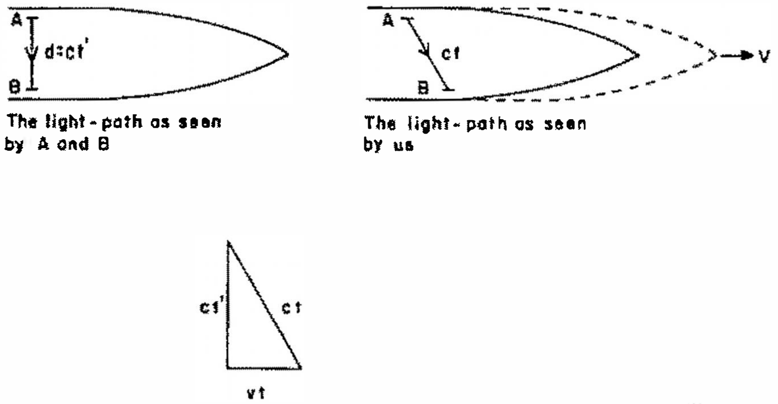


Figure 4.2

larger than that of a light-beam because of the ball's slower speed. To us the light travels farther than d, so it must take a longer time to make the trip, since light moves at the same speed in both frames.

If t is the time interval measured by us, the ship will move a distance vt in time t, which is the base of the triangle. The Pythagorean theorem then gives $(ct')^2 + (vt)^2 = (ct)^2$ which can be solved for t' to give

$$t' = t\sqrt{1 - v^2/c^2}. \qquad (4\text{-}1)$$

Therefore light takes a shorter time, by the factor $\sqrt{1 - v^2/c^2}$, to reach B as measured by A and B than it takes in our frame of reference. This is the same as saying that clocks on the ship (which are used by A and B to measure t') run slow compared to clocks in our frame of reference. For example, if the ship is moving past us at a velocity $v = \frac{3}{5}c$, and we measure a time interval to be one second, we will observe that the ship clocks advance by only $t' = t\sqrt{1 - v^2/c^2} = \frac{4}{5}$ second. The ship clocks are running slow from our point of view.

It is important to point out the care which would be required to verify this result. How do we measure the time interval in our frame of reference? We should not just sit back and watch A and B, starting and stopping our stop-watch when we see the signal sent and received. For A and B might not be at the same distance from us, so light from them

informing us of the sending and receiving would take different times to reach us. This would be an important effect, since the experiment already takes place at the speed of light. Therefore, just as in the length and time measurements discussed in Chapter I, it is necessary to have two observers in our frame of reference, one beside each event. The two observers have previously synchronized their clocks, and the one who is right beside A at the instant the signal is sent records this time as read by his own clock, while the one who is right beside B when the signal is received records the time as read by his own clock. They then compare notes, and the difference in their readings is what we mean by the time interval in our frame of reference.

The formula for time dilation is rather strange for clocks moving faster than the speed of light, since then the factor $\sqrt{1 - v^2/c^2}$ is an imaginary number. If such a clock reads a time t' represented by some real number, it would seem that our clocks should read a time t represented by an imaginary number. This is very difficult to interpret physically, so at least provisionally it would make sense to restrict ourselves to objects moving slower than c. In later chapters the reason for this restriction in relativity theory will become more physically clear. In particular, we will show that no finite force can make a particle move even as fast as light, that a particle moving faster than light would have an energy and momentum given by imaginary numbers, and that if a message were sent from one person to another faster than c, there would exist frames of reference in which the message was received before it was sent!

Needless to say, the experiment with the rocket ship and light-beam is just a thought experiment, which will probably never be done owing to the difficulty of making $\sqrt{1 - v^2/c^2}$ differ appreciably from unity. Fortunately, time dilation has been observed in a different way -- namely, in experiments on various kinds of unstable fundamental particles. Such particles can either be created naturally by cosmic rays hitting the atmosphere, or by using high-energy accelerating machines. Each species of unstable particle has a characteristic average lifetime, which can serve as a kind of clock whose rate can be measured as a function of the particle's velocity. For example, there is a particle called the μ-meson, or *muon*, which decays on the average in a time $T = 2.2 \times 10^{-6}$ seconds as measured by clocks in the muon's frame of reference. That is, if we measure the lifetimes of a large number of muons at rest in our frame of reference, their average lifetime will be about 2.2 microseconds. But if a muon moves past us, from our point of view its clock will run slow, so to us it will last *longer* than T before it decays. More precisely, if a large number of muons all move past us at some velocity v, *we* will find their average lifetime to be *greater* than 2.2 microseconds. From their *own* point of view, with measurements made in their own rest-frame, the muons will decay in their usual lifetime of 2.2 microseconds.

As an example, suppose a particular muon decays in 2.2 microseconds as measured by clocks in its rest-frame. If it moves by us at 4/5 the speed of light, which is not unusually fast for such particles, the dilation factor $\sqrt{1 - v^2/c^2} = \sqrt{1 - (4/5)^2} = 3/5$. Therefore, since $t' = 2.2$ microseconds, $t = \dfrac{t'}{\sqrt{1 - v^2/c^2}} = \dfrac{5}{3} \times 2.2$ microseconds. This particle lasts 67% longer than a similar particle at rest, as measured in our frame of reference, which means that it will be able to move farther than "expected" before it decays. This feature is very convenient in high-energy experiments, because it means that equipment can be spread out and made in larger sizes than would be necessary if the particles decayed sooner.

Muons are produced in great numbers in the upper atmosphere by the decay of particles called *pi-mesons*, or *pions*, which are themselves created in collisions of cosmic-ray protons with air molecules. If these muons actually decayed in their standard average lifetime of 2.2 microseconds from our point of view, they would almost all be gone before reaching the earth's surface. For example, a muon moving at nearly the speed of light would only move a distance of $cT = 3 \times 10^8 \times 2.2 \times 10^{-6} = 660$ meters, which is considerably less than the height of the atmosphere. A very large muon flux is nevertheless observed, since from our point of view they don't decay that fast. An experiment

has recently been carried out by Frisch and Smith* to test the time-dilation phenomenon quantitatively. They counted the number of muons at the altitude of Mt. Washington in New Hampshire (6265 feet) and compared this count with the number observed at sea level. Using only muons with speeds between .9950 c and .9954 c, they found by statistical methods that these muons lasted 8.8 ± 0.8 times longer than muons at rest. Theoretically, for muons of these speeds in their detection set-up, they calculated $\dfrac{1}{\sqrt{1 - v^2/c^2}} = 8.4 \pm 2$, so the time dilation effect is in good agreement with experiment.

In the *spaceship* experiment, we didn't specify what *kind* of clock was used by the observers. In fact, *every* moving clock, whether it is a wristwatch, radiating atom, decaying muon, heartbeat, hourglass, or whatever, must run slow. That is, since "time" runs slow, a sensible theory of physics must claim that the various clocks we use to *measure* time run slow when moving. This is easy to *say*, but it is often difficult to show in detail why particular clocks run slow, in terms of gears, pendulums, biological processes, and so on.

The simplest clock to analyze, to see how time dilation "works," consists of a mirror and a repeating flashbulb which emits a pulse of light every time light hits it. The clock will "tick" (the bulb will flash) with a time interval $\Delta t' = 2\,D/c$ when it is at rest with respect to us, as shown in Figure 4.3. This interval is the time it takes for the light from the flashbulb to bounce off the mirror and return.

Now suppose the clock moves to the right, or equivalently we run past the clock to the left. Then the light follows the path shown in the right-hand part of the figure, and obviously has to cover a greater distance between ticks.

From Pythagoras we have $\left(\dfrac{c\Delta t}{2}\right)^2 = D^2 + \left(\dfrac{V\Delta t}{2}\right)^2$

$$\text{or } \Delta t = \frac{2\,D/c}{\sqrt{1 - V^2/c^2}} = \frac{\Delta t'}{\sqrt{1 - V^2/c^2}} > \Delta t' . \qquad (4\text{-}2)$$

The clock runs slow by the expected factor.

* D.H. Frisch and J.H. Smith, *Am. Jour. Phys.* *31*, 342, 1963.

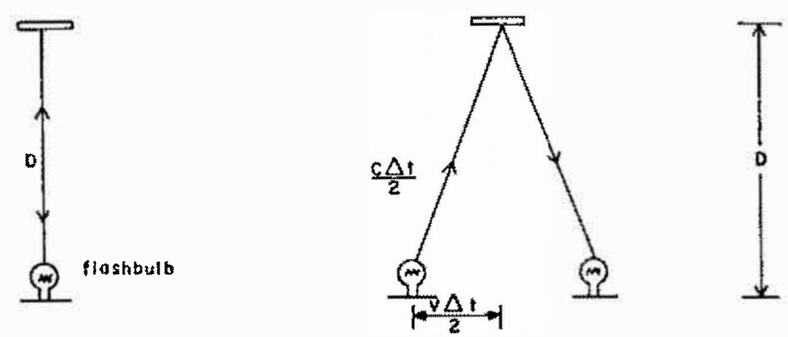


Figure 4.3

The frame of reference in which an object is at rest is said to be the "proper frame" for that object. The "proper time" for the object is the time read by a clock in the proper frame. Thus since a clock is itself an object, a clock measures its own proper time. The proper time between ticks of the flashbulb clock is 2 D/c, while the time between ticks is longer than the proper time when measured in any other frame of reference.

Time dilation is a basic "law of nature" which says that everything moving past you seems to age more slowly than it would at rest. A human being can be viewed as a clock which will "run slow" (age slowly) when moving past you. Your twin brother can travel to the star Sirius and back (a round-trip of 20 light-years) at 4/5 the speed of light, so that you would expect him to be $20/\frac{8}{10} = 25$ years older than when he left. Actually he will only be $25\sqrt{1-(4/5)^2} = 15$ years older. So when he returns, he'll be 10 years younger than you, since you've aged the full 25 years in the interim.

This situation gives rise to the so-called "twin paradox," which has generated a great deal of controversy. If your twin flies away and returns younger than you, why can't he turn the tables and claim that you left him and came back, implying that you should be younger? After all, from Einstein's postulate that absolute motion is undetectable, it shouldn't be possible to tell which twin is moving — everything is symmetrical.

As a first step in the resolution of this paradox, it should be realized that in fact the situation is definitely not symmetrical, and that it is possible to tell which twin went away and came back. In order to leave and return, somebody has to accelerate during a part of the trip. Both brothers will agree as to who accelerates, just by watching their accelerometers (pendulums, say). Einstein's results are only valid for observations made in unaccelerated reference frames, so that the advertised time dilation of moving objects is only true for the experiment as a whole if viewed by the twin who stays at home. We cannot analyze this situation using the time dilation factor if we move with the twin who leaves, because we would then be in an accelerated reference frame.

Using general relativity, or Einstein's theory of gravity, it is possible to analyze the world from accelerated reference frames. Then it is found that another effect, known as the gravitational red shift, slows down accelerated clocks, by just the right amount to give the same result we get by staying with the earthbound twin.* The twin who leaves and comes back (and therefore accelerates) is younger than the stay-at-home at the end.

REFERENCES

1. The experiments on muons to check time dilation are described in an article by Frisch and Smith in the American Journal of Physics 31, 342 (1963), and also by Rossi and Hall in the Physical Review 59, 223 (1941).

2. Another experiment related to time dilation is the measurement of the Doppler shift of light from moving sources, which is discussed in Chapter XII. References are listed there.

3. There are innumerable papers on the twin paradox. Some will be listed here, and others can be found in their references:

* See Appendix E, and for a more complete treatment see C. Møller's The Theory of Relativity, Oxford Press, 1952.

"Some Recent Experimental Tests of the Clock Paradox" by C.W. Sherwin, Phys. Rev 120, 7 (1960).

"The Clock Paradox in Relativity" by C.G. Darwin, Nature 180, 976 (1957).

"Relativistic Observations and the Clock Paradox" by J. Terrell in Nuovo Cimento 16 457, 1960.

"Relativity and Space Travel" by J.R. Pierce in Proc. I.R.E. 47 1053 (1959).

"The 'Clock Paradox' and Space Travel" by E.M. McMillan in Science 126, 381 (1957).

See also Appendix E.

PROBLEMS IV

1. A clock moving at speed $v = \frac{3}{5}c$ reads twelve o'clock as it passes us. In our frame of reference, how far away will it be when it reads one o'clock?

2. The mean lifetime of $\pi^{\pm}$ mesons is about 2.5×10^{-8} seconds in their rest-frame. If a beam of pions is produced which travel on the average 10 meters before decaying, how fast are they moving?

3. A spaceman with 50 years to live wants to see the Andromeda nebula (2 million light-years away) at first hand. How fast must he travel?

4. Two π^{+} mesons are created, one at rest in the laboratory, and the other moving at $v = \frac{4}{5}$ c. Each decays in 2.5×10^{-8} seconds in its own rest-frame. Find

   a. the lifetime of the moving pion as measured in the laboratory;

   b. the lifetime of the pion at rest in the laboratory as measured in the frame of the "moving" pion.

CHAPTER V
LENGTHS

AFTER FINDING that time has lost its absolute character, with clocks running at different rates when measured in different frames of reference, we had better be cautious about all kinds of things. Accepting the non-intuitive hypothesis that the speed of light is the same in all inertial frames, many other concepts should be reexamined. For example, might it not also be true that the measurement of distance depends on the observer's frame of reference?

A. Transverse Lengths

In looking back at the discussion of time dilation in the previous chapter, we find that it was actually assumed that distances weren't changed! More precisely, it was assumed that distances perpendicular to the direction of relative motion were the same to both observers, as for example in Figure 4.3, where it was taken for granted that the vertical distance between the flash-bulb and mirror was "D" in both frames of reference.

Fortunately, this assumption that transverse distances are unchanged is correct, as can be seen from a simple thought-experiment. Two men A and B are each equipped with a meter-stick having a thin knife-blade attached to one end, as shown in Figure 5.1. They run toward each other at a high relative speed, holding the sticks perpendicular to the direction of motion with the bottom end barely skimming the ground. If the sticks are really of the same length, the knives should hit each other, but if one stick is longer than the other, it will be sliced off by the knife on the shorter stick. Supposing that each man's stick is exactly one meter long to him, we would like to show that in fact the knives will hit each other, indicating that the stick moving past each man is also one meter long to him. This would prove that transverse lengths are unaffected by motion. That is, from A's point of view his own stick has a length of one meter, but he is not sure that a moving meter-stick (one meter as measured by B) will have the same length. We want to prove that in fact it is.

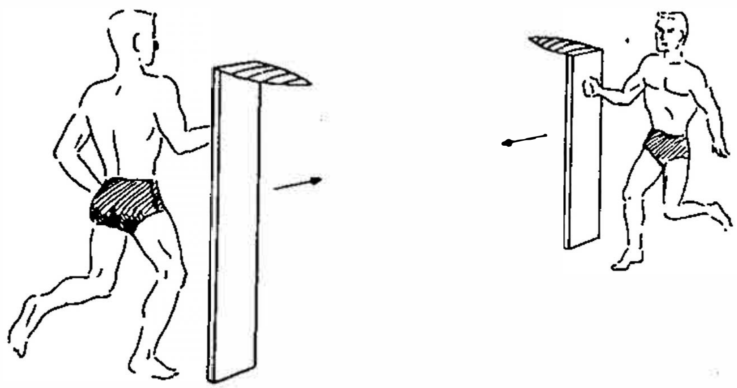


Figure 5.1

The proof follows by contradicting the other alternatives. First suppose that B's stick is shorter than one meter as seen by A. Then B's knife will slice off the top of A's stick. This fact doesn't depend upon who is observing it: it is definitely A's stick (and not B's) which has been cut off. The whole experiment was set up in a symmetrical way, playing no favorites between A or B, but it ends in an unsymmetrical way, with A getting his stick cut off. This can't happen, according to Einstein's first postulate, because it means there is an *a priori* reason for preferring one reference frame over the other. In such an originally symmetric experiment, with the laws of physics the same for both A and B, everything which happens to A should also happen to B.

The second alternative, that B's stick is longer than one meter as seen by A, implies that B's stick will be cut off, leading to the same contradiction. A preferred frame of reference could again be chosen. The remaining possibility is that the knives will hit each other, which is a symmetrical result, showing that to either observer the meter sticks have the same length. Therefore relativity agrees with our intuition that transverse lengths are unaffected by motion.

B. The Longitudinal Contraction of Lengths

In the situations we've discussed of decaying muons and moving twins, there lurks another effect, showing that longitudinal lengths

are affected by motion. We stand on the earth watching muons rain down, passing through several miles of atmosphere even though they ought to be able to go through only about 660 meters. This we interpret as a verification of Einstein's prediction that moving clocks run slow. A meson lasts longer than when it is at rest, which is why it can move so far. But what is going on in the muon's frame of reference? In its rest-frame, the muon decays in the standard time of 2.2 microseconds, so it can't possibly go several miles, even if it is moving at nearly the speed of light! Similarly, from the standpoint of the twin traveling to Sirius, why does the trip seem to take only a comparatively short time? Two possible explanations come to mind:

1. Velocities are not reciprocal – if we measure the velocity of someone with respect to us, he may find a different velocity of us with respect to him. Thus from his own point of view, the traveling twin may be going faster than 8/10c, and the muons may think they're going faster than light!
2. Distances are different in the two frames. A "moving" object may measure the distance it has to go to be less than the distance measured by a "stationary" observer. From the point of view of observers in the muon's frame of reference, the atmosphere would be very thin (< 660 meters high), and the moving twin would find the distance between the earth and Sirius to be less than 10 light-years, by just enough to allow him to complete the journey in only 15 years.

In other words, since by definition velocity = distance/time for any observer, if the time is different for two observers, the velocity and/or distance must be different also. We can't change our ideas about time without changing our ideas about something else also.

Clearly it is the first alternative which must be thrown out, since it contradicts Einstein's first postulate. If the relative velocity between the two objects depends upon which object was measuring it, we would have an absolute way of distinguishing between two frames of reference. We could say that one frame was "better," because the relative velocity was smaller, say, in that frame. This trouble shows up in an extreme form in the mu-meson experiment, where in the muon's frame it would have to move several miles at a speed greater than that of light. Air molecules would therefore be rushing past the muon with speed $v>c$, and their time would be contracted by the factor $\sqrt{1 - v^2/c^2}$, which would be an imaginary number.

According to the second alternative, which is the correct one, the distance of travel is shorter to the moving object. The distance from earth to Sirius as measured by the traveling twin is only $10\sqrt{1 - (8/10)^2} = 6$ light-years! Then, since $t = d/v$, the travel time will be shortened to him by a factor $\sqrt{1 - (8/10)^2}$, which we know to be the case. That is, he will explain the fact that he only requires 15 years to make the round-trip by claiming that the total distance is only $20\sqrt{1 - (8/10)^2} = 12$ light years. So $t = d/v = 12/(8/10) = 15$ years.

This effect is called the "Lorentz-Fitzgerald contraction," proposed independently by these two gentlemen to explain the Michelson-Morley experiment, but the idea was not completely understood and integrated with other relativistic effects until Einstein's theory appeared in 1905. Stately roughly, objects moving past us with velocity v are contracted in their direction of motion by the factor $\sqrt{1 - v^2/c^2}$. Equivalently, if we are moving past something, it is contracted by the same factor when measured in our reference frame. The atmosphere to the cosmic-ray muons is a very thin layer, so that they have plenty of time to penetrate it before decaying. The rest-length of an object is the length measured in the frame which is at rest with respect to the object. In any other frame, the measured length will be shorter than the rest-length. The fact that an object is largest in its rest-frame does not violate Einstein's first postulate, because it does not specify a preferred reference frame. It is true that an object's rest-frame could be taken to be a preferred frame for that object, but a different object might have a different rest-frame, so no overall preferred frame could be specified.

The Lorentz contraction is essential for understanding a "longitudinal flash-bulb clock." Figure 4.3 in Chapter IV showed the "transverse flash-bulb clock," which runs slow by the factor $\sqrt{1 - v^2/c^2}$ when moving. This was just a particular example of the general rule that any

clock must run slow by the same factor $\sqrt{1 - v^2/c^2}$ as it moves past an observer with relative velocity v. So now consider the same clock turned 90° with the light going back and forth along the direction of motion of the clock, as shown in Figure 5.2. When at rest, as shown in

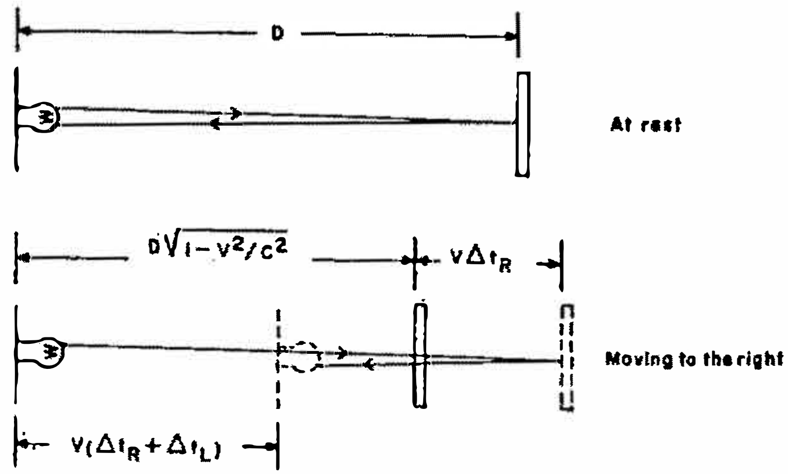


Figure 5.2

the top figure, the time between flashes is $\Delta t' = 2D/c$. When the whole apparatus moves to the right, as in the bottom figure, care is needed in calculating the total light travel-time. While the light moves to the right, before hitting the mirror, it must overcome the distance to where the mirror was when the flash bulb fired (the Lorentz-contracted distance $D\sqrt{1 - v^2/c^2}$), and also the distance the mirror moves in the meantime. If the light requires a time Δt_R to reach the mirror, the mirror will move a distance $v\Delta t_R$ during this time. The total distance the light has to travel to get to the mirror is then

$$c\Delta t_R = D\sqrt{1 - v^2/c^2} + v\Delta t_R,$$

$$\text{giving}\quad \Delta t_R = \frac{D\sqrt{1 - v^2/c^2}}{c - v}. \qquad (5\text{-}1)$$

On the return trip, if the light requires a time Δt_L to return to the flash-bulb, the flash bulb will move a distance $v\Delta t_L$ during this time. Therefore the total distance traveled by the light on the return trip is only

$$c\Delta t_L = D\sqrt{1 - v^2/c^2} - v\Delta t_L, \text{ so that}$$
$$\Delta t_L = \frac{D\sqrt{1 - v^2/c^2}}{c + v}. \quad (5\text{-}2)$$

The total time between ticks is therefore

$$\Delta t = \Delta t_R + \Delta t_L = D\sqrt{1 - v^2/c^2}\left(\frac{1}{c+v} + \frac{1}{c-v}\right) = \frac{2cD\sqrt{1 - v^2/c^2}}{c^2 - v^2} \frac{2D/c}{\sqrt{1 - v^2/c^2}} = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}. \quad (5\text{-}3)$$

This is the same result as for the transverse clock, namely that moving clocks run slow by the factor $\sqrt{1 - v^2/c^2}$. The Lorentz contraction was essential in getting this result, showing again that time dilation and length contraction are part of the same relativistic physics. You can't have one without the other.

We are dealing here with ideas that are not very intuitive, so it is necessary to take some care in describing the measurements necessary to see if a moving object is contracted. Because of the Lorentz-Fitzgerald contraction, we are tempted to say "anything moving looks shorter," which would seem to be a direct consequence. We could presumably see this effect if the velocity of light were small, say 10 meters/second. Then we might think an automobile or a bicyclist going down the street would look squashed up, and a spherical bowling ball would look like an ellipsoid. The joker is in the words "look" and "see," which we have used rather loosely, not being careful to note what kind of measurement the words imply. As a matter of fact, they imply a pretty unsatisfactory method of measurement, even in classical mechanics without any time dilation or length contraction.

As a particular example, suppose a railroad train is moving along a straight track at a velocity approaching that of light. It starts off to our left and moves past us to the right while we stand beside the track looking on. Now at any particular time, the view we get is the sum of all the light reaching our eyes at that time. A "view" is defined by the light simultaneously hitting our eyes. But since some parts of the

train are farther away from us than other parts, light will take longer to reach us from some parts than others. Therefore, as the train approaches, light from the caboose must have left before light from the engine did, so that our eyes will receive both light rays simultaneously. We see the caboose where it was a long time ago, whereas we see the engine where it was only a short time ago. But a long time ago the train was still far away, so the caboose will appear to be far away, even though the engine is close by! So as the train approaches it will actually look much longer than you might expect. By the same reasoning, it is easy to discover that as the train pulls by and rushes off in the other direction, it will look very short, even shorter than predicted by the factor $\sqrt{1 - v^2/c^2}$.

The apparent stretchings and squashings occur because of the finite speed of light. They would be observed for a fast train even if the world obeyed classical physics, without Einstein. The introduction of relativity has the effect of superimposing a Lorentz-Fitzgerald contraction on these other effects, so that for example a train right beside us (with the engine and caboose equally distant) will be shorter than its rest-length by the factor $\sqrt{1 - v^2/c^2}$.

We have reasoned here on the assumption that a train is essentially a one-dimensional object, and haven't worried about the effects of height and depth. It is very interesting to figure out the appearance of a three-dimensional body moving past, which is taken up in Appendix C. The important thing to remember here is that the Lorentz contraction is found by making simultaneous measurements of the position of the two ends, which is what we usually mean by measuring a length.

As an example of making a careful measurement, suppose we wanted to know the length of a rhinoceros charging rapidly past us. There are various ways we might make an experimental measurement:

1. If we knew the speed of the rhinoceros ahead of time, we could stand to one side with a stopwatch, starting it when the front end of the rhinoceros reaches us, and stopping it when the hind end passes, a time Δt later. We could then say that the length is $v\Delta t$

This won't work unless somebody has already measured his velocity, and the rhino has cooperated by maintaining it. If we are not so lucky, a better approach is method two, which needs more equipment and observers.

2. We can ask a friend, with whom we have synchronized watches (see Chapter VI on how to do that!), to stand a distance from us. If the front end reaches us just as the hind end passes our friend (i.e. our watches read the same), then the length is just the distance between us and our friend. This distance we can measure with a meter-stick any time, either before or after the experiment. The important thing is that we have measured the position of his two ends <u>simultaneously</u>. This has required <u>more than one observer</u>, so that there has been no problem in accounting for light-travel times between the object and the observer.

3. Another thing we might do is take a snapshot of the beast, and measure his size on the film. Taking account of the camera's magnification, and the lateral distance of the rhino from this camera, we can figure out his length. This would be equivalent to taking a quick look at him, noting the angle he subtends, and then computing his size by triangulation. This is obviously an example of a "phony" method, since it measures where his head was at one time and where his tail was at another. Using only a <u>single</u> observer to make a measurement means that light from various parts of the subject require different times to reach him, producing errors in figuring out lengths.

In the following chapters we will be asking how things look to various observers. This should be taken as a shorthand way of asking about the result of careful <u>simultaneous</u> measurements made in the observer's frame of reference, using in general <u>several</u> clocks and <u>several</u> observers in that frame of reference. Except in Appendix C, we won't discuss any more the odd effects brought about by light leaving different parts of an object at different times so as to reach a single observer simultaneously. Our measurements will be of type 1 or 2.

REFERENCES

For more on the visual appearance of moving objects, see Appendix C and references listed there.

PROBLEMS V

1. The disk of the Milky Way galaxy is about 10^5 light-years in diameter. A cosmic-ray proton enters the galactic plane with speed $v = .99$ c.
   a. How long does it take the proton to cross the galaxy from our viewpoint?
   b. How long does the proton think it takes?
   c. How wide is the galaxy to the proton (in its direction of motion)?

2. A spaceship of rest-length 100 meters passes by the earth at a speed such that only $\frac{1}{3} \times 10^{-6}$ seconds is required for it to pass by a given point, as measured by clocks on the earth.
   a. How fast is it moving?
   b. How long is the ship from the earth's point of view?

3. Electrons in the Stanford two-mile linear accelerator will reach a final velocity of about .9999999997 c. How long would the linac be to such an electron? Therefore how much time would it take to travel this distance to such an electron (assuming it were to move the whole distance at this velocity)? How long would it take the electron to make the trip as seen by Stanford?

4. The lifetime of the ρ-meson is about 10^{-23} seconds in its rest-frame. If the shortest distance that can be resolved in a photograph of a ρ production process in a bubble chamber is about 10^{-4} centimeters, how fast must a ρ go in order for us to see it? How far would the ρ think it had gone before decaying?

5. A hole in a table-top is 1" wide. A block of wood which is 1-1/2" wide in its rest-frame is shot along the table-top toward the hole at a velocity such that it is only 3/4" wide in the frame of the table. From the point of view of the table, the block should fall

through the hole. But in the frame of the block, the block is 1-1/2" wide and the hole is only 1/2" wide! So how can the block fall through the hole? How can this apparent paradox be resolved? (Appendix A presents a similar problem, also left for the reader to solve.)

CHAPTER VI
SIMULTANEITY

CONTRARY to classical physics and "common sense," the preceding chapters have shown that moving clocks run slow and that moving objects are contracted. But the job of demolition has only begun. Everything in physics must be viewed in the light of Einstein's postulates, either to be possibly modified or even rejected entirely. As we have seen, even concepts which Newton and others thought were a priori and absolute, like space and time, have had to be brought under physical investigation and changed. The topic to be discussed now is more upsetting to most people's intuition than any other conclusion of relativity.

A. The Relativity of Simultaneity

We will find in this section that simultaneity is relative. In other words, if two events are simultaneous in one frame of reference, they need not be simultaneous in some other frame of reference. Suppose for example that two supernovae are born in the universe in different galaxies. Does it always make sense to claim that supernova A blew up first, or would some observers claim that supernova B blew up first? We're not talking about the fact that somebody closer to B might see it explode first, simply because the light from the earlier explosion at A hasn't had time to reach him yet. We suppose that he will correct for this fact.

To answer the question whether simultaneity is absolute or relative, consider the following "experiment": we are calmly sitting in our spaceship in the midst of empty space, when suddenly two other (identical) spaceships approach from opposite directions and pass each other, as shown in Figure 6.1. Rocket "A" moves to the right, and rocket "B" moves to the left, with equal and opposite velocities as we watch them. Just as they pass, we fire bolts of energy

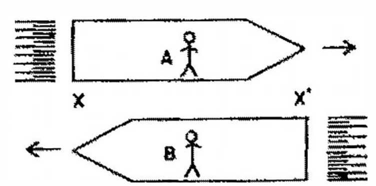


Figure 6.1.

at points x and x', which explode between the two ships just as the nose of one reaches the tail of the other. To us, both explosions happen at the same time, so we would say the two events are simultaneous.

But are they simultaneous to the inhabitants of A and B? Suppose there is an observer in the middle of each ship. Each observer knows he is in the middle of his ship, because he has carefully measured his position by using a meter-stick. First consider the man in A. During the time the light from x and x' moves toward him at velocity c, he has moved somewhat to the right, so he will actually see the explosion from x' before that from x. He can therefore say "I'm halfway between x and x', and I saw the light from x' first, so the explosion at x' must have happened earlier than the one at x". On the other hand, the observer on B moves to the left while the light is reaching him, so the light from x gets to him before the light from x', allowing him to say: "I'm halfway between x and x', and I saw the light from x first, so the explosion at x must have happened earlier than the one at x' ".

This result is easy to understand if we watch the whole experiment from the viewpoint of one of the other observers. As seen by the observer in B, rocket A is very short, so if explosion x happens when the nose of B is beside the tail of A, and if explosion x' happens when the nose of A is beside the tail of B, then the two events can't possibly be simultaneous as seen by B. Figure 6.2 shows the rockets in two positions as seen by B.

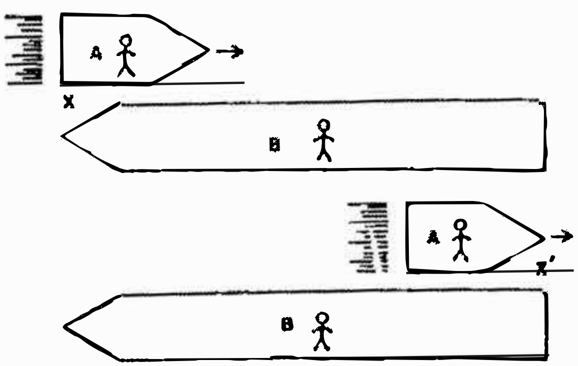


Figure 6.2

It is clear that B will claim that event x happened before event x'. In short, the question "which event really happened first?" will be answered differently by different observers. No over-all answer can be given. From A's point of view, explosion x' really happens before explosion x, B's ship is really shorter than A's, and B's clocks really run slow. He knows these things, because he has found them out by careful and well-defined measurements. But he would be cautious not to ascribe his reality to everybody, and would say only that certain facts are correct from his standpoint. From B's point of view, explosion x really happens before explosion x'. From our point of view, the explosions are really simultaneous, but we must admit that A and B have an equal right to do experiments and make conclusions from them, and that they will find the explosions are not simultaneous to themselves.

B. Clock Synchronization in a Single Reference Frame

The outcome of the spaceship experiment indicates that simultaneity is relative, and that clocks in one frame are not synchronized with those in another. In order to understand more clearly how this comes about, we will search for a satisfactory method of synchronizing two or more clocks in a single frame of reference, and then later show that these clocks will not be synchronized to observers in a different frame of reference.

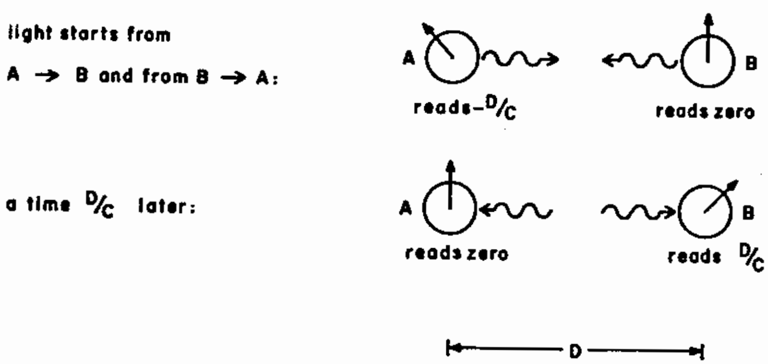
If we are presented with two clocks, at rest with respect to us and separated by a distance D, how can we synchronize them? We will try four different approaches, of which two will turn out to be satisfactory.

look over at clock and set same?

1. Let observers be put beside the two clocks A and B. A possible definition of synchronization to the observer beside A would be for both A and B to always read the same, as seen by him. That is, if he looks over at clock B, it will read the same time as his own clock A. The trouble with this definition is that if the clocks are set so that they look synchronized to the observer at A, they will not look synchronized to the observer at B. This definition neglects the fact that the light

from clock B requires a time $t = D/c$ to reach A, so that by the time a signal from B reaches A, clock B reads a <u>later time</u>, according to an observer at B. In fact, if observer A uses this definition of synchronization, in which both clocks read the same to him, the observer at B will see clock A lag <u>behind</u> clock B by a time $2D/c$, as illustrated in Figure 6.3. With this method, then, observers in the same reference frame

Synchronization according to method 1.



At the time D/c, the observer at A sees that clock A reads $t=0$, and that B reads $t=0$, since that is what B read when the light left it. By trial definition no. 1, clocks A and B are synchronized to the observer at A. But the observer at B sees that clock B reads $t=D/c$, and that A reads $t=-D/c$, which is what it read when the light left it. Therefore by definition no. 1, the observer at B will claim that the clocks differ by $\Delta t=2D/c$, and so are not synchronized to him. An unsatisfactory definition.

Figure 6.3

will disagree as to whether or not two clocks are synchronized, making the definition unsatisfactory.

Carry clocks away!

2. If we want to synchronize a lot of clocks, we could begin with them all together, and just set them to read the same. Then we might carry them out to various places, and by definition assert that they are all synchronized. The rather obvious problem with this definition is that the clock readings will depend upon exactly how the clocks are carried to their final locations. The time dilation effect will insure that all the clocks will run slow with respect to a stationary observer, but some will run slower than others, depending upon how fast they are carried, and also for how long a time they are carried. Another group of clocks, synchronized at a different position, and dispersed to the same locations as the first group, will generally disagree with the first group. This method is therefore also an unsatisfactory way of synchronizing clocks.

calculating looking!

3. Our next attempt to find a method of synchronizing two clocks will involve taking account of the time needed for signals to pass between them. Let us carefully measure the distance D between clocks A and B. As in the first method an observer is stationed beside each clock, and in addition each observer is equipped with a flash bulb which can be fired. They then agree on the following procedure: When clock A reads $t = 0$, observer A will set off his flash bulb. The flash will be seen by observer B, who will immediately set his clock to $t = D/c$, thus accounting for the light transmission time. Clocks A and B we claim are synchronized. This is an entirely consistent definition, because at any later time t another flash bulb can be set off by B whose light will reach A at $t + D/c$, which is what A's clock actually <u>will</u> read when he receives the light.

Halfway between!

4. A fourth approach to clock synchronization is the "halfway between" method, which is actually equivalent to method 3. We put two observers with clocks at A and B, measure the distance between them, and put a flash bulb at the halfway point.

Each observer having previously agreed to set his clock to $t = 0$ when the flash reaches him, the bulb is fired. The light will take equal time to reach A and B, so the observers will be justified in believing their clocks are synchronized. Given an additional flash bulb, the reader will be able to prove that methods 3 and 4 are equivalent, so either procedure can serve as a means of synchronizing two clocks.

Because of the lack of simultaneity in two different frames of reference, we have been very careful in our definition of clock synchronization. We've found that it is possible to synchronize two clocks in the same reference frame by a straightforward procedure. Since extreme care is required, it is also necessary to show that it is possible to synchronize three or more clocks in the same frame. Clearly if we can synchronize clocks A and B, it is also possible by the same procedure to synchronize clocks B and C. It is left for the reader to convince himself, using an actual experimental method, that if this is done, clocks A and C will be automatically synchronized as well. Therefore a well-defined means of synchronizing clocks can be developed, so that simultaneity is a meaningful idea in a single reference system. Two events would be simultaneous if the clocks placed beside them read the same when the events take place. Observers throughout a single spaceship (in the previous section) can synchronize their clocks, and agree whether or not two explosions occur simultaneously simply by comparing the readings of the two clocks on the ship, which are in proximity to the explosions when they go off.

C. In the Very Process of Synchronizing Two Clocks, a Moving Observer Disagrees

Suppose two clocks A and B, both in the same frame, are synchronized by method 4, the "half-way-between method." A flash bulb is set off half-way between them, the flash travels toward both clocks at speed c, and they are both set to $t = 0$ when the light reaches them. The process is shown in Figure 6.4. Now suppose we are at rest in a frame which is moving to the left at uniform velocity v. To us the

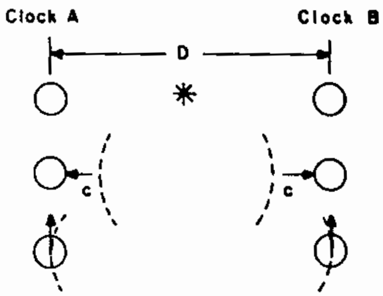


Figure 6.4

clocks move to the right, and the distance between them is contracted to $D\sqrt{1 - v^2/c^2}$. We want to watch, from our frame of reference, the process of synchronizing the clocks. Four stages in this process are shown in Figure 6.5. In our frame of reference, clock A will intercept the flash before clock B, because A is moving toward the light-source, whereas B is moving away from it. Therefore, since the reception of the flash is the cue for each clock to be set to $t = 0$, A will read ahead of B as seen from our frame of reference. Thus in the very process of synchronizing the clocks by the most reliable and well-defined method, they come out unsynchronized to us.

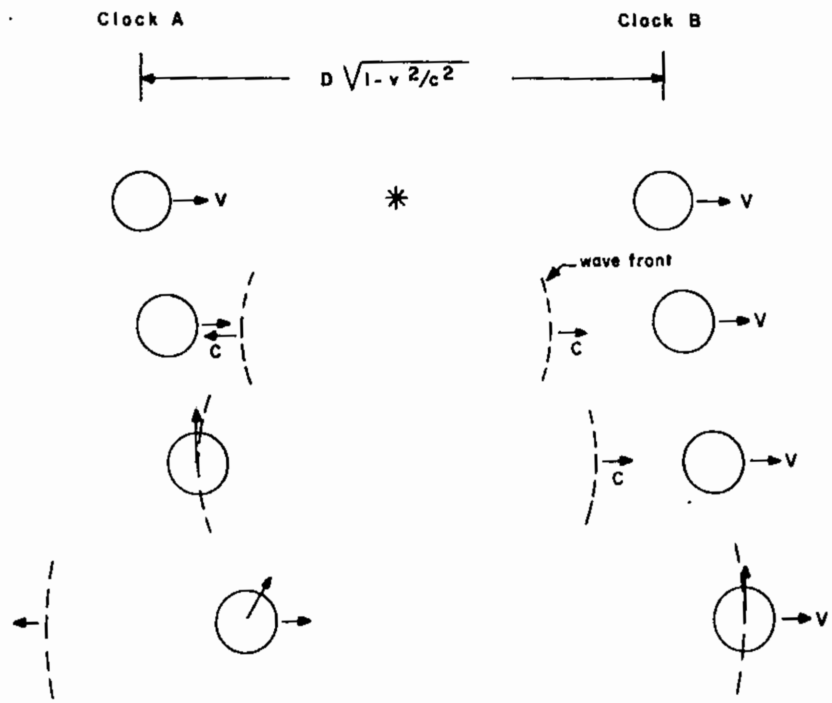


Figure 6.5

It is straightforward to calculate how much the two clocks will differ in our frame of reference. To begin with, suppose our clocks read $t = 0$ just as the bulb fires.* At stage three of Figure 6.5, when the light flash meets clock A, our clocks read $t = t_3$, given by

$$ct_3 + vt_3 = \frac{D}{2}\sqrt{1 - v^2/c^2}, \text{ or } t_3 = \frac{D\sqrt{1 - v^2/c^2}}{2(c+v)} \qquad (6\text{-}1)$$

found from equating the distance between clocks A and the flash bulb at the moment of firing to the sum of the distances traveled by the light and by clock A. At stage four, when the light reaches clock B, our clocks read $t = t_4$, given by

$$ct_4 = vt_4 + \frac{D}{2}\sqrt{1 - v^2/c^2}, \text{ or } t_4 = \frac{D\sqrt{1 - v^2/c^2}}{2(c-v)} \qquad (6\text{-}2)$$

found from equating the distance the light travels to the sum of the distance between clock B and the flash bulb at the moment of firing, and the distance traveled by clock B.

The time difference between stages three and four is then

$$\Delta t = t_4 - t_3 = \frac{Dv\sqrt{1 - v^2/c^2}}{c^2 - v^2} \qquad (6\text{-}3)$$

as measured by our clocks. But clock A runs slow to us by the factor $\sqrt{1 - v^2/c^2}$ during this interval, so to us will read

$$\Delta t' = \Delta t\sqrt{1 - v^2/c^2} = \frac{Dv\,(1-v^2/c^2)}{c^2 - v^2} = Dv/c^2 \qquad (6\text{-}4)$$

when clocks B read $t = 0$. In short, the clocks to us will be out of synchronism by an amount $\Delta t' = Dv/c^2$, with the chasing clock (A in Figure 6.5) reading ahead in time. Note that D is the rest-distance between the clocks, in their direction of motion. Clocks moving along side by side, neither chasing the other, will be synchronized in both frames.

* That is, our clocks have been previously synchronized, and our clock which is beside the bulb when the flash occurs reads $t = 0$.

D. A Rocket with Clocks

The results for the reading of clocks and meter-sticks arrived at so far lead to the following three rules:

1. Moving clocks run slow by the factor $\sqrt{1 - v^2/c^2}$.
2. Moving objects are contracted in their direction of motion by the same factor $\sqrt{1 - v^2/c^2}$.
3. Two clocks synchronized in their own rest-frame will not be synchronized in other frames, except in those special frames in which they are spatially separated only perpendicular to their direction of motion. The clock which chases the other will read ahead (show a later time) of the clock in front by an amount $\Delta t = Dv/c^2$, where D is the rest-distance between them along their direction of motion.

As an example of applying these results, picture a rocket of rest-length 100 meters moving by at a velocity $v = 4/5$ c. On the ship there are clocks at the nose and tail, labeled N and T, respectively, which have been synchronized. On the ground are three strategically placed clocks, labeled A, B, and C, which are synchronized in our ground frame of reference. To fix the zero of time, we suppose that our clock B and the clock N in the nose of the ship both read $t = 0$ just as they pass. At this instant, the situation to ground-observers is as shown in Figure 6.6. The rocket is only $100\sqrt{1 - v^2/c^2} = 100 \cdot \frac{3}{5} = 60$ meters long, and our clocks A, B, and C have been placed 60 meters

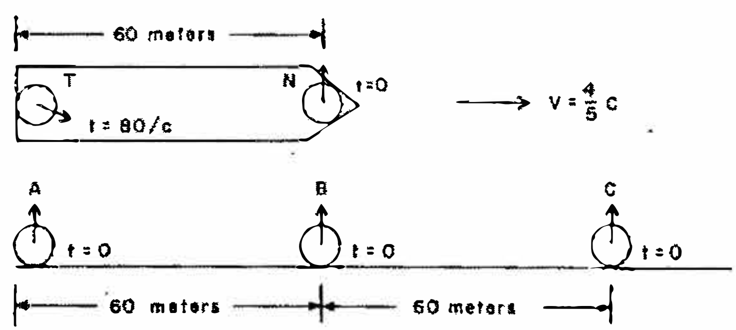


Figure 6.6

apart. By rule 3, clock T reads ahead of clock N by $\Delta t = \frac{vD}{c^2} = \frac{4}{5} \cdot \frac{100}{c} = \frac{80}{c}$ seconds, with c in meters/second.

Somewhat later the ship reaches clock C, and the tail passes clock B, requiring a travel time $t = \frac{\text{distance}}{\text{velocity}} = \frac{60}{\frac{4}{5}c} = \frac{75}{c}$ seconds. This will be what the ground clocks read, but those on the ship will run slow by the factor $\sqrt{1 - v^2/c^2} = 3/5$, so they will advance by only $\frac{3}{5} \times \frac{75}{c} = \frac{45}{c}$ seconds. Therefore the new situation to ground-observers will be as shown in Figure 6.7. Clock T still reads ahead of clock N by $\Delta t = 80/c$ seconds, as required by rule 3.

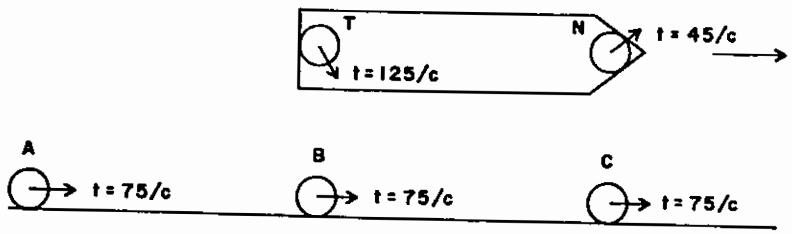


Figure 6.7

The important results are that

a. when clocks N and B pass, they each read $t = 0$ (by definition!),

b. when clocks T and A pass, they read $t = 80/c$ and $t = 0$, respectively,

c. when clocks N and C pass, they read $t = 45/c$ and $t = 75/c$, respectively, and

d. when clocks T and B pass, they read $t = 125/c$ and $t = 75/c$, respectively.

These facts can't depend on the frame of reference from which they are observed. For example, both people on the ground and people on the ship will agree that when clocks T and A pass, they read respectively $t = 80/c$ and $t = 0$. To convince yourself of this, imagine letting the clocks nick each other slightly as they pass, so that each stops running without being completely demolished. Each clock then reads

a definite time, and if this is twelve o'clock as seen from one frame, it will be twelve o'clock as seen from any frame.

To make sure that all these results are consistent, we can now try viewing the sequence of events from the standpoint of observers on the ship. To them, the rocket is at rest and has its full rest-length of 100 meters. The ground is rushing past to the left at $v = 4/5\ c$, and the three ground-clocks, instead of being 60 meters apart, will be only $60\sqrt{1 - v^2/c^2} = 60 \cdot \frac{3}{5} = 36$ meters apart. So when the nose of the ship and the middle ground-observer meet, the situation to people on the ship is as shown in Figure 6.8.

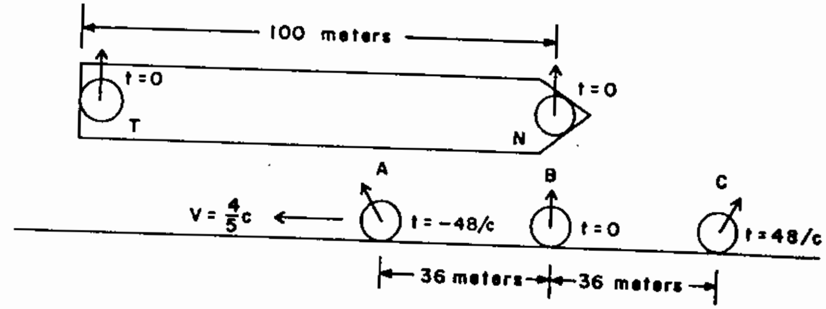


Figure 6.8

As before, clock N and clock B both read $t = 0$ as they pass, since this defines the origin of time for both systems. Clock T must also read $t = 0$, since it is synchronized with clock N on the rocket. The rest-distance between neighboring ground-clocks is $D = 60$ meters, so from rule 3 they will differ in time by $\Delta t = \frac{vD}{c^2} = \frac{4}{5}\frac{60}{c} = \frac{48}{c}$ seconds as seen from the ship. Note from the figure that clock C reads a later time than B, which in turn reads a later time than A, since the rule is that chasing clocks read ahead in time.

A while later, clock C passes clock N. This travel time will be $\Delta t = \frac{\text{distance}}{\text{velocity}} = \frac{36}{\frac{4}{5}c} = \frac{45}{c}$ seconds, which will be what the ship clocks read. Each ground-clock will run slow by $\sqrt{1 - v^2/c^2} = 3/5$, so each will advance by only $\frac{3}{5} \cdot \frac{45}{c} = 27/c$ seconds. The result is pictured in Figure 6.9. Notice that clocks N and C read $t = 45/c$ and $t = 75/c$, respectively, which is the same result found from the point of view of observers on the ground.

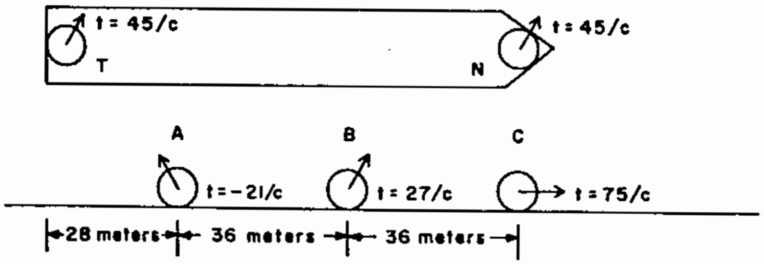


Figure 6.9

The next event takes place when clock A passes the ship's tail. This requires a travel-time $\Delta t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{28}{\frac{4}{5}c} = 35/c$ seconds. Then the ship-clocks will read $45/c + 35/c = 80/c$ seconds, and the ground-clocks will advance by an amount $\frac{3}{5} \cdot \frac{35}{c} = 21/c$ seconds, resulting in the situation shown in Figure 6.10. Note that clocks T and A read $t = 80/c$ and $t = 0$, confirming the result of the ground observers.

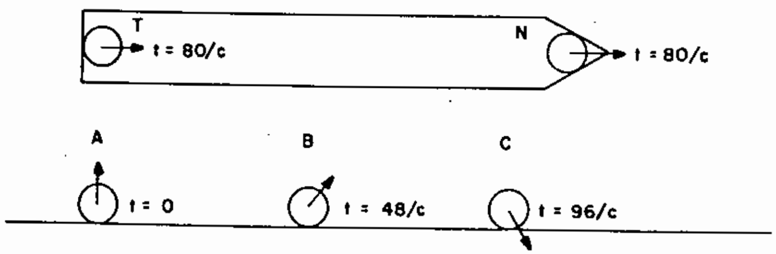


Figure 6.10

Finally, clock B will pass the ship's tail at a time $\Delta t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{36}{\frac{4}{5}c} = 45/c$ seconds still later, so the ship clocks will read $t = 80/c = 45/c = 125/c$ seconds, and each ground-clock will advance by $3/5 \cdot 45/c = 27/c$ seconds, as shown in Figure 6.11. Clocks T and B read $t = 125/c$ and $t = 75/c$, as previously found by observers on the ground.

What we have shown is that even though weird things go on, such as contracted lengths, dilated times, and lack of synchronization of moving clocks, certain facts are <u>independent</u> of the observer's frame

of reference. Namely, the readings of two clocks as they pass each other will be agreed upon by everybody.

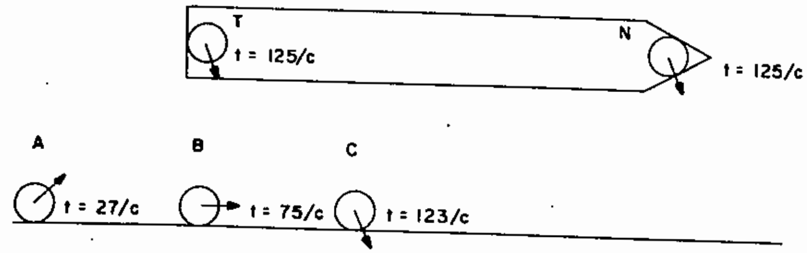


Figure 6.11

PROBLEMS VI

1. Two clocks have been previously synchronized in our frame of reference. We stand beside one, and look at the other, which is 30 meters away. What will it appear to read when the clock beside us reads $t = 0$? Now the distant clock is carried to us in 1 second, with velocity 30 meters/sec. By how much will the two clocks, now side by side, differ?

2. Believing that the sun is about to become a supernova, we blast off for Sirius. Just as the journey is half over, we see explosions from the sun <u>and also from Sirius</u> at the same instant! Are we justified in concluding that in our (the spaceship's) frame of reference the two explosions were simultaneous?

3. Show by outlining a conceivable experiment that if clocks A and B are synchronized, and if clocks B and C are synchronized, then clocks A and C will also be synchronized.

4. A rocket of rest-length 1000 meters moves with respect to us at $v = \frac{3}{5}c$. There are two clocks on the ship, at the nose and the tail, which have been synchronized with each other. We on the ground have a number of clocks, also synchronized with one another. Just as the nose of the ship reaches us, both our clock and the clock in the nose of the ship read $t = 0$.
   a. At this time $t = 0$ (to us) what does the clock in the tail of the ship read?